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1989 J. Phys.: Condens. Matter 1 7961

(http://iopscience.iop.org/0953-8984/1/42/018)

J. Phys.: Condens. Matter 1 (1989) 7961–7964. Printed in the UK

LETTER TO THE EDITOR

The Luttinger model with a finite-range potential

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Received 25 July 1989

Abstract. It is proved that the physics of the Luttinger model is insensitive to the variation of the potential range. New exact Luttinger Green functions are computed.

A long time ago, with the aim of describing the physical behaviour of a one-dimensional (LD) system of electrons, Tomonaga proposed a 'bifermionic' model [1], for which Luttinger developed a technically more convenient version [2]. The only condition imposed by Tomonaga for obtaining the bifermionic model (in fact, for finding simple commutation relations for the density operators $\rho^{(\pm)}$) was that the potential must be a long-range one; let us call it the Tomonaga restriction. Soon after, the Tomonaga–Luttinger (TL) model was diagonalised by Mattis and Lieb for any potential satisfying a condition imposed on its strength—the Mattis–Lieb restriction, independent of Tomonaga's. So, the TL is obtained as a relevant physical model if the Tomonaga restriction is satisfied, and can be solved exactly if the Mattis–Lieb restriction is fulfilled.

It is well known that the physical behaviour of the TL model may be obtained, to a certain extent, without making a particular choice of the potential [1, 3–5]. However, the most interesting information—the phase diagrams, for instance, which can be obtained if the correlation functions are known—requires making a certain choice for the potential, in order to find compact formulae. In fact, to avoid exceedingly large computational difficulties, a contact potential is chosen (a delta function)—and so a zero-range one.

It is natural to ask whether this choice, being in contradiction with the Tomonaga restriction, does not destroy the relevance of the results for describing the physical behaviour of the system. In other words, we may ask ourselves if the physical behaviour of the model is sensitive or not to the variation of the potential range.

We shall prove that the answer is negative—in the following way. We shall choose an arbitrary-range potential, compute the corresponding Green functions and demonstrate that they have the same singularities as the Green functions corresponding to a contact potential. This conclusion may be extended to all correlation functions, at $T = 0$ or $T > 0$. Also, we shall prove that these conclusions hold for all the physically interesting potentials.

0953-8984/89/427961 + 04 $02.50

Let us consider a Luttinger model in which the undressed Fermi velocity is v_F, and the potential has the form

$$V_a(x) = V(\sin ax)/x \qquad a > 0. \tag{1}$$

Its range is characterised by the parameter a, and

$$\lim_{a\to\infty} V_a(x) = \pi V\delta(x) \tag{2}$$

$$\lim_{a\to 0} V_a(x) = 0. \tag{3}$$

The Green functions (generally, the correlation functions) may be easily evaluated using the bosonisation method [6, 7]. Let us introduce the notations

$$r_0^{(\pm)} = a[\mathrm{i}(\operatorname{sgn} t)(x - v_F t) - \alpha] \tag{4}$$

$$r^{(\pm)} = a[\![\mathrm{i}(\operatorname{sgn} t)\{x \pm v_F[1 - (V/2v_F)^2]^{1/2} t\} - \alpha]\!] \tag{5}$$

$$s = \tfrac{1}{2}\{[1 - (V/2v_F)^2]^{-1/2} - 1\} \tag{6}$$

where α is an infinitesimally small parameter (the cut-off parameter of the Luther–Peschel–Mattis bosonisation scheme [6]). Following the standard methods [8], we find (see [9] for details)

$$\begin{aligned} G_1(x,t) &= -\mathrm{i}\langle T\psi_1^+(x,t)\psi_1(0,0)\rangle \\ &= G_1^{(\delta)}(x,t)\exp(\mathrm{Ei}(r_0^{(+)}) - \mathrm{Ei}(r_0^{(-)})) \\ &\quad\times \exp[s^2(-2\mathrm{Ei}(-\alpha a) + \mathrm{Ei}(r^{(+)}) + \mathrm{Ei}(r^{(-)}))] \end{aligned} \tag{7}$$

where

$$\mathrm{Ei}(-y) = -\int_1^\infty \frac{\mathrm{e}^{-xy}}{x}\,\mathrm{d}x \qquad \operatorname{Re} y > 0 \tag{8}$$

and $G_1^{(\delta)}(x,t)$ is the Luttinger Green function corresponding to the delta potential obtained from (1) for asymptotic values of the parameter a, according to (2).

From general considerations,

$$G_2(x,t) = G_1(x,-t). \tag{9}$$

In spite of the daunting appearance of (7), it is clear that, as the Ei functions have no singularities, G_1 and $G_1^{(\delta)}$ have the same pole structure. It is easy to see that any correlation function may be cast in a similar form:

$$C(x,t) = C^{(\delta)}(x,t)\,f_{\text{reg}}(x,t) \tag{10}$$

where $C^{(\delta)}$ is the correlation function for the TL model with a potential obtained according to (2) and f_{reg} is a regular function—more exactly an exponential similar to that appearing in (7).

Although (10) was obtained at $T = 0$, it is clear that a similar computation at $T \neq 0$ [8] leads to the same factorisation, for any correlation function. So, equation (10) maintains its form for $T > 0$, too.

Similar results may be obtained for another potential with a simple Fourier transform:

$$U(x) = U\delta(x) + V(\sin ax)/x. \tag{11}$$

Putting

$$g = (1/2v_F)(U/\pi + V) \qquad G = U/2\pi v_F \tag{12}$$

$$s = \tfrac{1}{2}[(1-g^2)^{-1/2} - 1] \qquad S = \tfrac{1}{2}[(1-G^2)^{-1/2} - 1] \tag{13}$$

$$\omega = v_F(1-g^2)^{1/2} \qquad \Omega = v_F(1-G^2)^{1/2} \tag{14}$$

$$r^{(\pm)} = a[\mathrm{i}(\operatorname{sgn} t)(x \pm \omega t) - \alpha] \qquad R^{(\pm)} = a[\mathrm{i}(\operatorname{sgn} t)(x \pm \Omega t) - \alpha] \tag{15}$$

we find for the Luttinger Green function corresponding to the potential (11):

$$G_1(x,t) = G_1^{(\delta)}(x,t)\exp[(1+s^2)\mathrm{Ei}(r^{(+)}) + (1+S^2)\mathrm{Ei}(R^{(+)}) + s^2\mathrm{Ei}(r^{(-)}) - S^2\mathrm{Ei}(R^{(-)}) + 2(S^2 - s^2)\mathrm{Ei}(-a\alpha)] \tag{16}$$

i.e. an expression similar to (7). In particular, equation (16) confirms and generalises in all orders the results of Theumann [10] referring to the analytic dependence of G_1 on the parameter V from (11).

Any realistic potential may be approximated as a linear combination of functions like (11)—which is evident in the Fourier space. So, the same computational scheme may be developed for any potential and an equation like (10) remains valid, in the general case.

The fact that some major aspects of the physical behaviour of the system are independent of the particular form of the potential was pointed out from the beginning by Tomonaga [1]. Gutfreund and Schick [4] demonstrated that the shape of the momentum distribution function is only determined, at $T \simeq 0$, by the $q = 0$ Fourier component of the potential. Here we have generalised such conclusions for the entire physical behaviour of the model and for any temperature.

As is well known, the first Luttinger Green function was computed by Theumann [10]. The expressions (4) and (16) obtained here are probably the only Luttinger Green functions for which compact formulae may as yet be obtained, at least using the bosonisation technique.

We can sum up this Letter as follows. The TL model describes well a LD metal for long-range potentials; but the physics of the model is the same, irrespective of the potential range; so, any potential may be used in specific calculations.

These considerations support the idea that, in many-body problems, the form of the two-body potential may be surprisingly irrelevant.

The author is indebted to Dr M Apostol and Dr I Bulboacă for useful discussions and comments.

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